

# Beethoven in the Temperaments

## Historical Tunings on the Modern Concert Grand

These notes are taken from Enid Katahn's CD "Beethoven In The Temperaments — Historical Tunings on the Modern Concert Grand." The two Well Temperaments employed on the piano were in use between 1750 and 1825, a brief, but very significant period in the history of keyboard composition.

### Temperament

"Temperament" is both the process and result of making slight changes to the pitches of a pure musical scale, so that the octave may be conveniently divided into a usable number of notes and intervals. In the history of Western music's 12-note octave tradition, there have been several, fundamentally different forms of temperament applied to the developing keyboard. Music suffers when performed in a tuning that is different than that which the composer used in its creation.

### Technology & Time

Since antiquity, musical temperament has been a topic of debate among instrument builders, theorists, musicians, and composers. The Aristoxineans argued fine points with followers of Pythagoras in 400 B.C., marking a beginning for twenty-three centuries of controversy over exactly what the relationships of notes to one another should be.

Evolving from Greek scales based on numerology and the appeasement of deities, to math-based systems of modern times, intonation and temperament in Western music have undergone continual change in response to advancing technology. At each stage of this musical history, composers used whatever resources were available to create their music. From the first hole drilled in a caveman's bone flute, to Pythagoras's ratios, to the computer synthesizers of today, the musical scale has reflected whatever technological advances musicians found useful.

As an example, development of the pipe organ's secondary wind chests did away with the short gusts of bellows in favor of a wind reservoir. This provided a new steadiness of air pressure, which led directly to greatly increased control over pitch. As more

notes could be carved out of an octave with this new found precision, additional raised keys were added between the existing notes on keyboard instruments. Musical composition took on additional harmonic complexity during this time, as composers discovered ways to utilize the new resources.

These developments occurred before the 14th century ended, finalizing the arrangement of the 12 notes-per-octave keyboard as we know it today. Tuning offered new challenges to create as many harmonious combinations as possible out of a larger, more complex group of notes. There were some very discordant sounds created in order to provide beautifully harmonious combinations elsewhere on those keyboards, and it took a musical ear to judge how the instrument sounded best.

Technology created musical complexity and the need for changing intonation in later years, as well. The modern brass orchestra became feasible only after 1840, when machines capable of making consistent valves were invented. Large groups of fixed pitch instruments, like brass and woodwinds, needed a tuning system that everybody could use, which added new demands for a “universal” tuning. By the end of the 19th century, the old ways of just tuning by listening were coming to an end.

## **Tonality Lost**

The rapidly developing technology of the 1800’s had a profound effect on the tuning of keyboard instruments. The changes in temperament were, to a large degree, in response to improved metallurgy. More power came from tighter wires, and as the demand for more powerful instruments grew, cast iron was employed to withstand the added strain. String tension and audible overtones increased dramatically.

Areas of dissonance, in the earlier temperaments of harpsichords and lightly strung fortepianos became excruciating on the overtone-laden, high tension pianos used by the Romantic composers. To mask this unavoidable dissonance the musical community became even more willing to accept an averaged system of tuning, one which sacrificed some harmony in all keys, so that none were excessively dissonant. Science was in fashion at this time and the new, “mathematically correct” tuning was easily embraced as a long sought ideal.

Looking back, we see that, to a degree, all temperaments are era-specific. The Gregorian and “plain-chant” musicians prior to 1200 A.D., without chromatic keyboards, were well served by the long-lived Pythagorean intonation using pure fifths. Early Baroque composers wrote within the restrictions of Meantone tuning. The Classical masters had various Well Temperaments from which to choose, before the Romantics forced the tonal issue and our modern age of Equal Temperament arrived. Late-20th century synthesizers and computers have now provided microtonalist composers of today with temperaments that are impossible on virtually any other instrument.

There is still disagreement over the dates of Equal Temperament's acceptance. There is no definitive answer, but there is considerable evidence that the final development of true Equal Temperament coincided with the Industrial age, the world's infatuation with science, the increasing consistency of musical instruments, and the development of the modern piano.

Thus, Equal Temperament, an ideal that had been theorized about for centuries, finally evolved to provide the atonal base and seamless modulation for the Romantic composers as well as our own 20th-century music. It is this relatively recent development in tuning history that defines our present-day keyboard sound.

However, music composed in another temperament era needs those era-specific tonal resources to display its full character. Something is lost when the temperament is radically changed from that which the composer used, to something that is an "average" for convenience. Something important is lost when a Beethoven piano sonata is performed on an equally tempered keyboard. That something is "The Character of the Keys", also called, "key colors."

## **Overtones, Ratios, and Intervals**

Musical pitch is most easily defined by numerical means, i.e., the number of times a string makes a back and forth movement, (a "cycle"), per second. The more cycles per second, the higher the pitch. The term "cycles per second" has been replaced by Hertz, abbreviated "Hz."

The pitch of A-440 means that the A string vibrates at the frequency of 440Hz. On the piano this would be A4 (or A49, A above middle C). The A string an octave above (A5 on the piano) vibrates at 880Hz and the A below (A3) corresponds to 220Hz.

Since musical pitch is measurable by number, the relationship between two pitches may be viewed as the ratio between two numbers; i.e., if two strings are vibrating at exactly the same speed, their ratio is 1:1. If one of these strings is caused to vibrate twice as fast, the ratio becomes 2:1. There is an infinite number of combinations between these two ratios, but only a few of them are used in western music.

The distance between two frequencies is referred to as an interval. In the above example, the 2:1 ratio describes the interval of an octave, which is just a doubling of pitch. The term, "interval" is used interchangeably to describe the musical combination of two notes, or the distance between them.

These ratios describe more than the relationship of the notes' fundamental vibrations. Ratios also describe frequency relationships that exist well above the pitches of the notes. To understand how this is so, it is necessary to understand some basics of string behavior.

A vibrating string produces a complex combination of simultaneous musical frequencies, called “partials” or “overtones.” It is as though there is an entire chord produced that we hear as a single pitch corresponding to the string’s lowest frequency. This lowest frequency is the string’s “fundamental,” or “first partial.”

The other partials sounding from this string are all higher frequencies than the fundamental, but, they are not random. They are specific pitches arrayed in an order representing whole number multiples (2, 3, 4, 5, 6, ...) of the fundamental vibration speed.

If, for example, a string has a fundamental of 100Hz, there are also frequencies being simultaneously produced that are multiples of this rate, i.e., 200, 300, 400, 500Hz, etc. The 500Hz frequency is the fifth “partial” of this note.

The pitches resulting from this progression form the intervals of the harmonic series by the following method:

1. The second partial (200Hz) is double the 100Hz fundamental; it represents a 2:1 ratio, (an octave).
2. The third partial (300Hz) is above the second partial (200Hz) by a ratio of 3:2 (a fifth).
3. Between the fourth and third partials, we find a ratio of 4:3, (a fourth), and between the fifth and fourth partial, we see the ratio of 5:4, (the major third).

As we continue going up, defining the pitches of the harmonic series by adding 100Hz to each successive partial, the intervals continue to decrease in size, since each additional 100Hz means an ever smaller percentage of the total vibrations occurring. Thus, the sixth and fifth partials are separated by 100Hz difference, just as the first and second partials are; however, the 600Hz and a 500Hz pitch, played together, form the musical interval of a Minor third. This is a much smaller interval than the octave span that separates the first two partials.

The progressively decreasing size of the intervals between partials insures that when two separate notes are played together, somewhere in their respective sequences of partials there will be a frequency that is common to both. This is important because it is by these “shared” higher partials that the tonal “natures” or “colors” of intervals, and the chords constructed from them, are formed.

The octave has always been recognized as special. Octaves share more partials than other intervals—the octave’s partials meet at several ratios, 2:1, 4:2, 6:3. Since the octave is the largest basic interval, it represents a harmonic unit within which all other intervals used in Western music are contained. Notes above an octave are repeats of the lower notes contained within, merely doubled in pitch.

Pythagoras (550 B.C.) demonstrated that a harmonious interval results from two pitches which have a ratio of 1.5:1 (more commonly written 3:2). For reasons still debated, this ratio is almost universally considered pleasing to “Western” musical ears. The 3:2 ratio fitted nicely with the prevailing mysticism of the time, and the Pythagorean scale was formed by using multiples of this ratio.

The building of scales with these ideal ratios began a system of tuning (Pythagorean tuning) that lasted longer than any other, from 500 B.C. to 1300 A.D. During this period, technology made little progress that was of use to musicians. The wind-blown organ continued its slow development, but it was still a crude instrument in 900 A.D.

Shown below are the fundamentals and partials of a Pythagorean fifth, the next most basic interval after the octave. Notice which partials in each overtone series find a matching partial from the other note.

[BUILD MISSING DIAGRAM]

The Major 3rd has its shared partial higher in the harmonic series. If in addition to our original 100Hz note, another note is played that has a higher fundamental of 125Hz, its partials are located at 250, 375 and 500 Hz. We see that both notes produce a common frequency of 500 Hz in their overtones (the fifth partial of the lower note, and the fourth partial of the upper note). Played together, these two notes have a ratio of 5:4, and form the musical interval of a major third (Maj3rd).

Musical intervals corresponding to the ratios of 4:3 (fourth), 5:3 (sixth), and 6:5 (minor 3rd) were, in time, all found to be pleasing, and as progress was made in the control of pitch, useful for composition.

These ratios describe ideal relationships. In reality, tuning requires that many intervals (other than the octave), be compromised.

## Purity and the Wolf

Intervals may exist in one of two states: Just, or tempered. If an interval is tuned so that the common partial is exactly the same from both notes, the interval is called “Just” (also called “Pure” by some, as it relates to purity of harmony). This state represents the height of harmony in the physical sense and is the most “in-tune” that the interval can be.

If one note of the interval is tuned slightly higher or lower from this “pure” (Just) state, the interval is said to be “tempered.” When an interval is tempered, the common overtone is no longer common, and a very important acoustical phenomenon occurs: a phenomenon that is the crux of tonal “color.”

Consider our example of the Just third, with its two notes at 100Hz and 125Hz. If we alter this interval by raising the upper note’s fundamental to 126Hz, its fourth

partial then becomes 504Hz. When combined with the 500Hz fifth partial of the lower note, these two mismatched partials begin interfering with each other, and produce a wavering, or “vibrato” effect of 4 cycles per second. This wavering is called “beating,” and is a defining characteristic of the interval’s musical nature.

The beating of tempered intervals is audible on the piano. Playing the C-E Maj3rd will produce beating that is heard two octaves above the E. This is the location of the fourth partial of the E and the fifth partial of the C, (see above illustration). If middle C is used, in an equally tempered tuning, this beating is quite fast. Playing the third an octave lower will decrease its beating speed.

Listeners, on first hearing, often refer to beating as a “fluttering” or a “tremolo” effect. Some say it sounds as though they are listening through a slowly turning fan.

The fifths are altered much less than the thirds in modern tuning. As a result, the shared partials of a fifth beats so slowly that they are barely audible. If one chooses to listen for the beating of a fifth, it occurs at a pitch one octave above the upper note.

Anytime two notes have shared partials whose frequencies are close, but not exactly the same, there will be beating in the interval. The speed of the beating is equal to the difference between the two notes’ (almost common) partials, and it is this speed that determines the musical “nature” of the interval. When the interval is Pure or Just and there is no beating, the general feeling is one of harmony. As the notes are tuned further apart, they are said to be tempered. As the tempering increases, the beating speeds up, and the feeling of harmony evolves into something more stimulative. When beating becomes too fast, the interval is heard as “out of tune,” or as a “wolf interval.” (Lore has it that the musical term, ‘wolf’, originated in medieval Europe, as the horribly out of tune intervals in some of the earliest keyboard tuning schemes evoked the howling of wolf packs.)

Between Purity and the wolves, there is a wide variety of tonal expression available; expression that is determined by how the interval is tempered.

## Tempering

So, why not tune all intervals purely, and be done with it? This is impossible because Just intervals do not fit evenly into the octaves if there are only twelve notes within the octave’s span. Pythagoras had demonstrated mismatches between multiples of simple, perfectly tuned intervals and octaves. One major discrepancy he found was the misfit between fifths and octaves, now called the Pythagorean Comma.

On a keyboard, the comma is heard as the difference between seven Just octaves versus twelve Just fifths, i.e., if seven just octaves are tuned one from another, up from the low A, and then 12 Just fifths are tuned from the same low A, the upper

results will be two very different A's. The A reached by the series of fifths will be much higher. Numerically, this can be shown as  $2^7 = 128$  (for the octaves), and  $(3/2)^{12} = 129.74$  (for the fifths). On the 12 note-per-octave keyboard, one note must represent both of these pitches for A, so there must be a compromise. Since fifths are too big to fit, some, or all of them must be made smaller, i.e., they are made smaller by flattening their upper note, or sharpening their lower note. The same problems affect all the other intervals.

Octaves do not sound good when compromised, so the fifths must be made smaller than Just if they all are to fit evenly. This enforced departure from perfectly matched partials is called tempering, and is responsible for tonal "color."

The effect of tempering is more pronounced in the thirds. It can be shown, mathematically, that the span covered by three Just Maj3<sup>rd</sup>s cannot equal one octave:

- The ratio of the Just Maj3<sup>rd</sup> is 5:4. On the keyboard, three contiguous Maj3<sup>rd</sup>s span the distance of one octave. If we multiply the ratios, we see that  $5:4 \times 5:4 \times 5:4 = 125:64$ . This is smaller than the ratio of the octave, which is 128:64 (i.e., 2:1).
- Since three Just Maj3<sup>rd</sup>s are not as wide as an octave, we must enlarge one, two, or all three of them to fill the octave's span. The Maj3<sup>rd</sup> begins to beat when widened (by lowering the bottom note or by raising the top note). This beating is important.

Thomas Young, reading to the Royal Society in January of 1800, stated the basic problem of temperament (note that he reverses nomenclature in his ratios, but they are still the same intervals):

*"It would have been extremely convenient for practical musicians...if three times the ratio of 4 to 5 (three Maj3<sup>rd</sup>s), or four times that of 5 to 6 (four Min3<sup>rd</sup>s), had been equal to the ratio of 1 to 2 (the octave). As it happens to be otherwise, it has been much disputed in what intervals the imperfection should be placed."*

The Maj3<sup>rd</sup>s are primarily responsible for the tonal "character" of the triad, and Baroque and Classical music are to a considerable degree, built upon the concept of the triad. Thus, the manner of tempering the Maj3<sup>rd</sup>s plays a critical role in the tonal structure of Baroque and Classical keyboard music.

## Choices

For the last century, the topic of temperament has been relegated to the "tall weeds" in the field of musical discussion. However, recent research now strongly indicates that modern tuning is quite different from that used in Beethoven's time. As a

consequence, a Beethoven piano sonata played in Equal Temperament is fundamentally different from the same music played in a temperament of his period, regardless of whether the instrument used is a fortepiano or a modern concert grand piano.

A basic understanding of tuning is needed to see how temperament affects keyboard music. In general terms, keyboards can accommodate the discrepancies arising from the commas by being tuned in one of three very different ways.

## **1: Meantone Tuning**

The notes can be tuned so that some key signatures (henceforth referred to as “keys”), have chords that are maximally in-tune, while other keys are based on chords that are terribly out-of-tune.

For example, in a C-to-C octave, the C-E third and the E-G<sup>#</sup> third may be left Just if the last third, A<sup>b</sup>-C, (actually G<sup>#</sup>-C), is sacrificed to the wolves and widened by the full comma. In this way, two thirds of the combinations are Just at the expense of the last, which becomes the “wolf” interval. The keyboard composers of the early-Baroque era composed with this tuning style, now referred to as “Meantone” tuning.

During Meantone’s reign, the wolf was accepted as being an imperfection of the instrument, and was therefore banished to the remote, less-played key signatures. The presence of these harsh, out-of-tune sounding combinations made Meantone a “restrictive” tuning, because there are some places that a composer cannot musically “go.” As a consequence, keyboard composers prior to 1700 were careful to not modulate through enough keys to encounter the wolf, preferring instead to restrict their music to more harmonious keys.

Meantone tuning sounds much more harmonious than what we are accustomed to hearing from keyboards today, but only in certain keys. The rest are usually considered unmusical except for special effects of jarring dissonance.

## **2: Equal Temperament**

Today’s Equal Temperament is a mathematically defined system of tuning in which all notes are equally spaced, like bricks in a wall. This is done by multiplying the frequency of each note by the twelfth root of two, in order to find the next note above. This process is logarithmic and places every note equidistant from its neighbors. Each fifth ends up being tempered one twelfth of the Pythagorean comma.

In contrast to Meantone, Equal Temperament makes no tempering distinctions between keys. The “wolf” is spread equally between all combinations, hence, all keys have the same amount of tempering, and there is no “tonal color” in one key that does not exist in the others. One could say that all keys are equally dissonant in Equal Temperament. Unlike Meantone, no key is more harmonious than any other.

This equality is obtained by making every like interval, other than the octave, equally “out-of-tune.” If one assigns an emotional value to a particular key in Equal Temperament, it is a conditioned, psychological response, as all keys are tempered alike and differ only in their pitch center. (There may be associations made with keys by their pitch, but that is a separate discussion from tempering.)

### **3: Well Temperament**

“Well Temperament” is the name used by J. S. Bach to refer to a tuning approach in which some keys are more in tune than others. All are musically usable, but contain varying degrees of the “wolf.” There were many Well temperaments in use between 1700 and 1825.

Well Temperaments attempted to address the problem posed by temperaments such as Meantone, in which some of the less-played key signatures were musically dissonant and harsh. This tuning approach was popular between the Early Baroque and Romantic eras. Although this was a brief, transitional stage in temperament history, it cannot be overlooked because of the music composed during this period.

## **Well Tempering and Key Character**

The defining characteristic of any Well Temperament is the tonal variety that exists among the keys. The comma is neither condensed into a few combinations nor spread evenly among all; rather, it is dispensed into the various keys in differing amounts.

This allotment of dissonance was the subject of intense debate among the theorists and musicians of the 18th and 19th centuries, yet there was a common form to virtually all Well Temperaments. The common form was that the “all white note” keys of C major and A minor (with no accidentals in the key signature) contained the most harmoniously tuned Maj3<sup>rds</sup>, far more in tune than our modern Equal Temperament. The other “simple” keys such as G, F, or E minor were slightly tempered. Keys with yet more accidentals (requiring the use of more black notes), absorbed greater amounts of the commas, causing the dissonance to increase in those keys. Hence, there was a range of harmony and dissonance available to the composer.

Associations between emotional response and musical harmony are very old; they were discussed in ancient Greece. Certain tunings (modes), were considered war-like, others were felt as peaceful. Some tunings, according to Plato, should not be heard by developing young minds, while exposure to others was considered essential to the full development of one’s potential.

By Beethoven’s day, the concept of “Key Character” (in which different keys conveyed specific emotional impressions) was much refined. A widely read and influential list of keys and their affective qualities, written by Christian Friedrich Daniel Schubart and

published posthumously in 1806, contained fashionable descriptions for all major and minor keys. For example:

*“C minor. Declaration of love and at the same time the lament of unhappy love. —All languishing, longing, sighing of the love-sick soul lies in this key.”*  
*“E major. Noisy shouts of joy, laughing pleasure and not yet complete, full delight lies in E major.”* *“C# minor. Penitential lamentation; ...sighs of disappointed friendship and love lie in its radius.”* *“C major is completely pure. Its character is: innocence, simplicity, naivety, children’s talk.”*

These florid descriptions were for audiences that expected and wanted to be emotionally moved. Modern sensibilities don’t provide the context for quite that much scenery in music today, and modern atonal tuning has provided little key character for at least a century. However, we can still be affected by the composer’s use of dissonance and harmony, if the contrasts are there.

In a significant number of people, purely tuned (consonant) intervals cause physiological reactions indicative of a sedative, restful response. These reactions include changes in heart rate and respiration, pupil dilation, etc. Conversely, highly-tempered (dissonant) intervals tend to cause stimulative effects. This suggests that differing levels of physical consonance and dissonance can be valuable tools for a composer intent on eliciting emotional responses from a sensitive audience.

Accepting that any specific emotional “meaning” that a musical key represents is an ineffable quality (since “meaning” is subjective and depends on far more than the tuning), it is still obvious that Just intervals have a different musical nature and effect from tempered ones. The harmonic journey that is the piano sonata allows ample opportunity for a composer to juxtapose musical tension with points of rest; to artfully move the listener from edgy anticipation to welcome tranquility. This is tonality at work.

## Historical Temperaments Today

Today’s modern concert grand is a wonderful instrument for the tonal music of 1800’s Vienna. It provides a wider range of expression and power than its predecessors. As a consequence, its use with the “colors” of Well Temperaments and the music of a bygone era that was created with them, produces dazzling effects in the modulatory passages. The flowing changes of musical tension are enhanced by the contrast of dissonance and consonance. These are musical effects that the composers certainly were aware of, effects that are not possible in Equal Temperament.

The “acceptable” amount of tempering has changed over the course of history, as royal courts and audiences have demanded different music at different times. In the Meantone era, when music was heavily influenced by the church, a good major third was a pure third and everybody knew where the wolf lurked. Today, with the

ubiquitous use of Equal Temperament, the lack of pure intervals (and therefore contrast in the keys) has deprived us of the depth that was written into the tonal music of the masters. In the Well Temperaments of the Classical Era, created during a time when art, science, and religion battled for dominion, we hear both the pure and the dissonant, the calm and the storm. In the characters of the keys, we hear the souls of the classical composers.

